Int. J. Heat Mass Transfer. Vol. 10, pp. 1023–1040. Pergamon Press Ltd. 1967. Printed in Great Britain

RELATIONSHIPS BETWEEN BUBBLE FREQUENCY, DEPARTURE DIAMETER AND RISE VELOCITY IN NUCLEATE BOILING

H. J. IVEY†

(*Received* 3 *October* 1966 *and in revised form* 28 *January* 1967)

Abstract—Many analyses of boiling heat transfer include the terms bubble frequency f, bubble diameter D, and bubble rise velocity u, in their expressions for nucleate boiling heat flux and critical heat flux. In order to simplify these expressions individual relationships between f and D, and the relationship $u = fD$, are often used over the entire range of bubble diameters.

Evidence is presented which indicates that a single relationship only approximately correlates f with D for all nucleate boiling bubble diameters. Three separate regions are suggested by the experimental data, which correlate the data for each region better than a single relationship: (a) hydrodynamic region, in which the major forces acting on the bubble are buoyancy and drag: $fD^{\frac{1}{2}} = 0{\cdot}90\ g^{\frac{1}{2}}$; (b) transition region, in which buoyancy, drag and surface tension forces are comparable: $fD^{\frac{3}{4}} = 0{\cdot}44\ g^{\frac{1}{2}}$ ($\mathrm{cm}^{\frac{1}{4}}$); and (c) thermodynamic region, in which conditions during bubble growth predominate: $fD^2 =$ constant. Theoretical explanations for the variation of f with D are proposed for regions (a) and (c).

Experimental observations indicate that the relationship $u = fD$ is only approximately valid over the range of bubble diameters: $0{\cdot}02 > D > 3$ cm. The relationship $u/fD = 1$, therefore, should not be incorporated into accurate theoretical analyses of the boiling phenomenon, in which the proportionality constant relating dimensionless groups is obtained as an exact theoretical value.

NOMENCLATURE

A,	area [cm^2];
a/g,	dimensionless acceleration;
b,	thermal boundary-layer thickness [cm];
C,	specific heat at constant pressure [J/g$_m$ degC];
C_D,	drag coefficient;
D,	bubble diameter [cm];
$\bar{D}$,	mean (arithmetic, unless otherwise stated) of D [cm];
D_c,	cavity diameter [cm];
f,	frequency of bubble emission [s^{-1}];
f_i,	bubble frequency at individual site [s^{-1}];
$\bar{f}$,	mean (arithmetic, unless otherwise stated) of f [s^{-1}];
G,	mass velocity of vapour in departing bubbles [g$_m$/cm^{2}s];
g,	acceleration due to gravity [cm/s^2];
g_c,	mass-acceleration/force conversion constant [g$_m$ s^2/dyn cm];
h,	heat-transfer coefficient [W/cm^2 degC];
k,	thermal conductivity [W/cm degC];
m,	mass of bubble g$_m$];
N,	bubble sites per unit area of heater surface, $= n/A$[cm^{-2}];
N_{Nu},	Nusselt number;
n,	number of bubble sites on heater surface;
Q,	heat energy [J];
q,	heat flow [W];
q/A,	heat flux [W/cm^2];
$(q/A)_{cr}$,	critical heat flux [W/cm^2];

---

† Senior Scientific Officer, Science Research Council, London, U.K.

R,	bubble radius [cm];
r,	radius [cm];
T,	temperature [°C];
t,	time [s];
t_b,	period of single bubble $= t_c + t_d$[s];
t_c,	contact time of liquid with heater surface for single bubble [s];
t_d,	growth time of bubble on heater surface [s];
u,	bubble rise velocity [cm/s];
V,	bubble volume at departure [cm^3];
V_i,	bubble volume at departure for a given site [cm^3];
$\bar{V}$,	mean (arithmetic, unless otherwise stated) of V_i [cm^3].

Greek symbols

α,	thermal diffusivity [cm^2/s];
δ_L,	thickness of superheated liquid layer [cm];
ΔT,	temperature difference [degC];
ΔT_c,	critical bubble superheat [degC];
$\Delta\rho$,	$\rho_L - \rho_v$ [g$_m$/cm^3];
θ,	wall temperature minus saturation temperature [degC];
λ,	latent heat of vapourisation [J/g$_m$];
μ,	viscosity [cm^2/s];
ρ,	density [g$_m$/cm^3];
σ,	surface tension [dyn/cm];
$\phi(\ldots)$,	function of $(\ldots)$.

Subscripts

b,	bulk liquid;
L,	liquid;
s,	saturation;
v,	vapour;
w,	wall of heater.

Superscripts

*,	surface averaged value;
**,	time and surface averaged value;
x,	exponent.

1. THE RELATIONSHIP $f = \phi(D, \ldots)$

1.1 *Introduction*

THE EXPRESSION relating frequency f to diameter D for nucleate boiling bubbles is of major importance in the analysis of boiling heat transfer. This arises because either bubble diameter or frequency may be eliminated in analytical expressions for heat transfer in both the nucleate boiling region and at the critical heat flux.

The product fD^3 occurs in almost every analysis of nucleate boiling to date. Examples of nucleate boiling theories which involve f and D are those of Jakob and Linke [1], Rohsenow [2], Sterman [3] and Treshchov [4]. These theories derive heat flux as the enthalpy transported by one bubble, multiplied by the frequency of bubble emission and the number of bubbles emitted per unit area. The heat flux associated with the formation of the bubbles is given by the following expression in which mean values of the variables are used:

$$\left(\frac{q}{A}\right)_{\text{bubble}} \propto \frac{n}{A}\bar{f}\bar{D}^3(\lambda\rho_v + c_L\rho_L\Delta T). \qquad (1)$$

The product fD^3 is also used in all analyses of the critical heat flux in pool boiling using a liquid continuous–vapour discontinuous type of model [6], and in other analyses using rather different models. Examples of such analyses using a liquid continuous–vapour discontinuous model are those of Deissler [7], Rohsenow and Griffith [8], Griffith [9] and Chang and Snyder [10], and an example using a different model of the disposition in space of the liquid and vapour phases is that of Zuber [11].

As Rallis and Jawurek [12] have pointed out, care should be taken in the definition of the mean values of the variables used in equation (1). In particular the mass velocity G of vapour leaving the heating surface is

$$G = \frac{1}{A}\sum_{i=1}^{n}(f_iV_i)\rho_v = \frac{n}{A}\bar{f}\,\bar{V}\rho_v. \qquad (2)$$

Most theories employ arithmetic means for both $\bar{f}$ and $\bar{V}$ for substitution into the right hand side of equation (2). This practice is in general invalid, because as soon as the mean value of either $\bar{f}$ or $\bar{V}$ is defined, the remaining parameter ($\bar{V}$ or $\bar{f}$) is also defined through equation (2). It may be noted that the f_i and V_i in equation (2) are themselves time-averaged mean values for the ith nucleating site although Rallis and Jawurek do not take this into account. Care is therefore necessary in defining f_i and V_i to evaluate the product $f_i V_i$ in the left-hand side of equation (2).

The ideal information to be derived from an experimental determination of f, V and n/A would appear to be the value of each individual bubble volume V with its associated frequency f (presumably measured as the inverse of the bubble period t_b) for each of the n nucleating sites. These values should then be measured continuously over a sufficiently long period of time to allow for any long-term variations in bubble production at any one site to be determined. This is a very difficult task and all experimental data for f and V have usually been in terms of their arithmetic averages for each boiling condition or, at most, for individual nucleating sites. The results obtained by the present author are time-averaged values of f and D for individual nucleating sites.

1.2 *Previous work on the relationship between bubble frequency and diameter*

The most clearly defined properties of a boiling bubble are its size at departure and its frequency of emission from a given nucleation site. Since Jakob and Linke [1] noted a similarity in the magnitudes of the product fD for vapour bubbles in water and in carbon tetrachloride, many attempts have been made to generalise the relationship between f and D. In general there are three different approaches to the problem: (a) hydrodynamic, (b) experimental and (c) thermodynamic. The three methods of approach are examined in the present section in order to compare their predictions (in Section 1.3) with a wide range of experimental data. It is also demonstrated that no single relationship applies over the entire range of D. Care should therefore be taken to choose a relationship between f and D most suited to the boiling conditions anticipated, when incorporating such a relationship in an analysis of boiling heat transfer.

(a) *Hydrodynamic region.* In this region the diameter and frequency are assumed to depend solely on the buoyancy and drag hydrodynamic forces acting on a bubble (thus inertia, surface tension, and viscous forces are neglected).

Cole [13] equated drag force with buoyancy force for a freely rising vapour bubble and, with the assumption of Deissler [7] that critical heat flux occurs when successive bubbles leaving the surface touch and coalesce at $fD = u$, deduced that:

$$fD^{\frac{1}{2}} = \left(\frac{4g(\rho_L - \rho_v)}{3C_D\rho_L}\right)^{\frac{1}{2}}. \tag{3}$$

It was observed experimentally that $C_D \simeq 1$ for steam bubbles rising at atmospheric pressure [13], and for $\rho_L \gg \rho_v$

$$fD^{\frac{1}{2}} = 1{\cdot}15\, g^{\frac{1}{2}}. \tag{4}$$

Zuber [11] incorrectly quoted Peebles and Garber's results, for the region of bubble size in which bubble rise velocity is independent of bubble diameter (see Appendix 1), as

$$u = 1{\cdot}8\left(\frac{gg_c\sigma(\rho_L - \rho_v)}{\rho_L^2}\right)^{\frac{1}{4}}. \tag{5}$$

Zuber assumed $2fD = u$, from the observations of Jakob [16] that an adhering bubble's centre of gravity rises with the same velocity as a detached bubble, and that $1/f$ is approximately half the bubble lifetime, which gave:

$$fD = 0{\cdot}59\left(\frac{gg_c\sigma(\rho_L - \rho_v)}{\rho_L^2}\right)^{\frac{1}{4}}. \tag{6}$$

McFadden and Grassmann [17] assumed that fD is dependent upon diameter and that $fD = \phi(\rho_L, \sigma, \Delta\rho, D)$. Dimensional analysis was stated to give the following equation (although g had not been included originally):

$$fD^2\left(\frac{\rho_L}{g_c\sigma D}\right)^{\frac{1}{2}} = \phi\left(\frac{g\Delta\rho D^2}{g_c\sigma}\right). \tag{7}$$

Primarily due to their experimental results for boiling nitrogen, and by assuming $\Delta\rho \simeq \rho_L$, they obtained:

$$fD^{\frac{1}{2}} = 0{\cdot}56g^{\frac{1}{2}}. \tag{8}$$

This is very similar to the Davies and Taylor [18] result for large spherical cap bubbles in which $u = 0{\cdot}472\ (gD)^{\frac{1}{2}}$, and where $u = fD$ is assumed.

What is believed to be a more accurate dimensional analysis of the hydrodynamic problem is given by the present author in Appendix 2.

It is of interest to note that the result of Zuber [equation (6)] is equivalent to that of Cole [equation (4)] and of McFadden and Grassmann [equation (8)], through use of the Fritz expression for bubble diameter at break-off

$$D \propto \left(\frac{g_c\sigma}{g\Delta\rho}\right)^{\frac{1}{2}}. \tag{9}$$

Equations (6) and (9) give

$$fD^{\frac{1}{2}} \propto \left(\frac{g\Delta\rho}{\rho_L}\right)^{\frac{1}{2}} \tag{10}$$

which is equivalent to the result of Cole, and of McFadden and Grassmann, because for most conditions $\Delta\rho \simeq \rho_L$.

(b) *Transition region.* In this region the bubble diameter and frequency are assumed to depend on three dynamic forces which are all similar in magnitude: these are the forces of buoyancy, drag, and surface tension. Analysis of these forces is difficult, and the variation of f with D is therefore obtained by experiment.

Jakob and Linke [1] proposed a boiling heat-transfer mechanism in which the parameters of importance were the ratio of surface area occupied by vapour bubbles to the total heater surface area, the ratio of bubble volume at break-off to that at the free liquid surface, and an expression for the rate of vapour formation from unit area of the heater surface. Dimensional analysis then gave

$$N_{Nu} = \phi\left(\frac{q/A}{\lambda\rho_v(fD)}\right). \tag{11}$$

From experimental observations on water and carbon tetrachloride it was found that $fD \simeq$ constant.

Nishikawa and Urakawa [19] made the following independent observations from their experimental data:

$$\begin{aligned} q/A &\propto (n/A)^{\frac{1}{2}} \\ h &\propto (fD^3n/A)^{\frac{1}{3}} \\ \theta &\propto (q/A)^{\frac{2}{3}}\,(n/A)^{-\frac{1}{6}}. \end{aligned} \tag{12}$$

Gaertner and Westwater [20] also observed experimentally the first of the expressions in equation (12). From equation (12) it may be deduced that fD^3 = constant.

Rallis and Jawurek [12] observed that in boiling from a thin horizontal wire to water at saturation, fD^3 = constant at any one particular heat flux. In addition, the value of fD^3 increased appreciably with increase in heat flux.

Siegel and Keshock [21] carried out experiments on bubble behaviour at reduced gravities and found that independently $f \propto a/g$ and $D \propto (a/g)^{-\frac{1}{2}}$. When combined, this gives the expression $fD \propto g^{\frac{1}{2}}$.

(c) *Thermodynamic region.* In this region the bubble diameter and frequency are assumed to be governed solely by thermodynamic considerations. For example as for conditions during bubble growth, where growth rate is determined by the heat-transfer rate through the region near the liquid–vapour boundary of the bubble.

Zizina-Molozhen and Kutateladze [22] assumed a steam bubble under thermal equilibrium; where q and h refer to the liquid–vapour bubble surface:

$$q = 4\pi R^2\rho_v\lambda\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right) = 4\pi R^2h\theta. \tag{13}$$

With $h\theta = q/A$, an average constant value $\overline{q/A}$ was assumed. Equation (13) was integrated between limits $t = 0$ and $t = t_c$, where t_c is the time the bubble exists on the heating surface, and gave

$$\frac{D}{t_c} = fD = \frac{2\overline{q/A}}{\lambda \rho_v}. \qquad (14)$$

Staniszewski [23] used equation (13) and an equation giving the heat transferred across the bubble wall, together with $(\mathrm{d}T/\mathrm{d}r)^{*}_{(r=R)} = \phi(R)$, to give (with no interval between bubbles departing in period $t_b = 1/f$):

$$\frac{fD}{2} = \frac{R}{t_b} = -\frac{k_L}{\lambda \rho_v}\left(\frac{\mathrm{d}T}{\mathrm{d}r}\right)^{**}_{r=R}. \qquad (15)$$

Hatton and Hall [43] started with the bubble growth equation developed by Plesset and Zwick [44] to give the diameter of the bubble at $t = t_d$:

$$t_d = \frac{\pi\alpha}{3}\left(\frac{D\lambda\rho_v}{4k_L \Delta T_c}\right)^2 \qquad (16)$$

where ΔT_c is the critical bubble superheat. Using the Hsu [45] and Han and Griffith [46] expression for bubble nucleation when the superheated liquid layer surrounding a critical bubble is above the temperature required for the initiation of its growth, Hatton and Hall assumed $\delta_L \gg D_c$ and for saturation $T_b = T_s$, which gave

$$\Delta T_c = (T_w - T_b) = (T_w - T_s) = \frac{4\sigma T_s}{\lambda \rho_v D_c}. \qquad (17)$$

Hatton and Hall used equation (24) and by putting $t_c = 0$ for all but very low pressures obtained $f = 1/t_d$. Combination of equations (16) and (17) then gave:

$$f = \frac{1}{t_d} = \frac{3}{\pi\alpha}\left(\frac{16 k_L \sigma T_s}{(\lambda\rho_v)^2 D D_c}\right)^2. \qquad (18)$$

For a given liquid and pressure fD^2 = constant. D_c^{-2}. The experimental data of Hatton and Hall are correlated by the expression $fD^2 = 4{\cdot}85 \times 10^{-7}$ (f in s^{-1}, D in cm), in which the value of the constant would be expected to increase for higher heat fluxes.

The present author suggests an alternative thermodynamic approach which gives f and D as functions of physical properties of the boiling liquid. For very small bubbles the bubble diameter is in the same order of magnitude as the thermal boundary-layer thickness. A model for the bubble growth cycle may be proposed as follows. At the beginning of the bubble cycle, cool bulk liquid fills the space left by the previously departed bubble, and a thermal boundary layer develops by heat conduction. This is a cyclic phenomenon which lends itself to solution by the heat-conduction equation for the quasi-stationary case, where the heating surface–liquid interface temperature changes periodically with time at a frequency equal to that of the bubble emission frequency. Approximating the liquid to an infinitely thick wall, and assuming that the interface temperature changes according to a sine-shaped curve of frequency $1/t_b$, then Eckert [5] noted that the temperature oscillations in the liquid have almost ceased at a liquid depth b given by

$$b = 1{\cdot}6\,(\pi\alpha t_b)^{\frac{1}{2}}. \qquad (19)$$

Equation (19) provides an expression for the maximum cyclic thermal boundary-layer thickness b.

Substituting thermophysical properties for nitrogen at saturation temperature into equation (19) and noting that $t_b = 1/f$:

$$f^{\frac{1}{2}} b = 1{\cdot}6(\pi\alpha)^{\frac{1}{2}} = 0{\cdot}086\ \mathrm{cm\,s^{-\frac{1}{2}}}. \qquad (20)$$

This may be compared with the equation of the line through the experimental results of McFadden and Grassmann for liquid nitrogen shown in Fig. 1 and evaluated at normal gravity:

$$f^{\frac{1}{2}} D = 0{\cdot}270\ (\mathrm{cm\,s^{-\frac{1}{2}}}). \qquad (21)$$

The similarity of equation (20) with equation (21) is evident. The not unreasonable proposal that the thermal boundary-layer thickness b is one third of the bubble diameter D (for these results) then gives $b = D/3$ and equivalence of equations (20) and (21). From this it may be deduced that $fD^2 = 23\ (\pi\alpha)$. Griffith [25] has predicted,

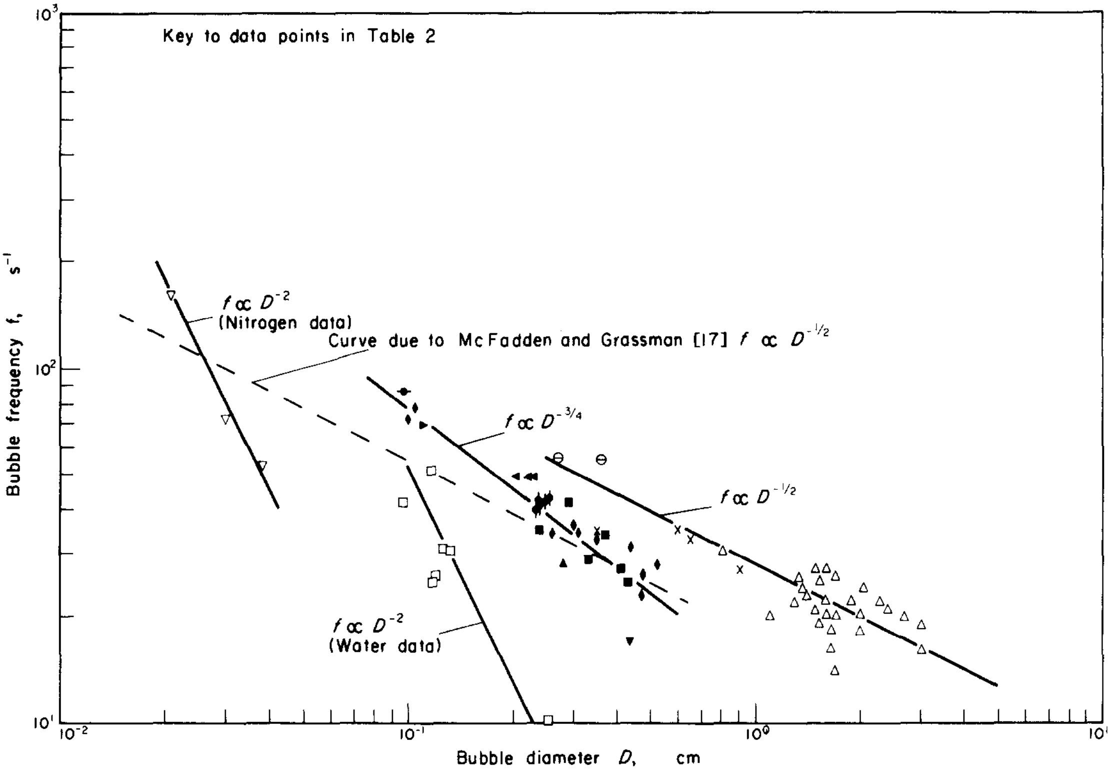

FIG. 1. Experimental data from the literature with the three proposed correlations.

from a bubble growth-rate model for small values of parameter $C_M = \rho_L C_L \Delta T/\rho_v \lambda$, that the maximum size attained by the bubble is proportional to the thickness of the superheated thermal boundary layer next to the heater surface. This is similar to the author's model, whereby the oscillation thermal boundary-layer thickness determines the frequency-diameter behaviour of the boiling bubbles for small bubbles. Han and Griffith [46] have also recently extended the use of the transient thermal boundary-layer thickness $b = (\pi \alpha t)^{\frac{1}{2}}$ to an analysis of the mechanism of heat transfer in nucleate pool boiling.

1.3 *Comparison of theoretical predictions with experimental data*

In the present section the theoretical predictions available from the literature are compared with experimental data, for the purpose of evaluating which theories are in reasonable accord with this data.

It is clear from Section 1.2 that a large variation exists in the published functional relationships for the expression: $f = \phi$ (D, a/g, system parameters, thermophysical properties). A comparison of the different relationships is given in Table 1, where the exponent of D varies from $-\frac{1}{2}$ to -3, four of the relationships involve system acceleration a/g, and five involve various thermophysical fluid properties.

The f and D data for nucleate boiling from the literature is plotted in Fig. 1. The key to the individual data points is given in Table 2, which also gives the literature source, type of liquid, and the percentage of the critical heat flux (where known).

The quantities f and D are dependent variables

Table 1. *Comparison of the expressions for bubble frequency as a function of diameter, system variables, and thermophysical properties. Frequency is proportional to the product of the parameters raised to the powers listed in the table*

Group	Author	Reference number	D	$\left(\frac{a}{g}\right)$	Other variables	λ	σ	ρ_v	ρ_L	k_L	C_L
Hydrodynamic (theories)	Cole	13	$-\frac{1}{2}$	$\frac{1}{2}$							
	McFadden and Grassmann	17	$-\frac{1}{2}$	$\frac{1}{2}$							
	Zuber	11	-1	$\frac{1}{4}$			$\frac{1}{4}$		$-\frac{1}{4}$		
Transition (experimental results)	Jakob and Linke	1	-1								
	Siegel and Keshock	21	-1	$\frac{1}{2}$							
	Rallis and Jawurek	12	-3								
	Nishikawa and Urakawa	19	-3								
Thermo- dynamic (theories)	Zizina-Molozhen and Kutateladze	22	-1		$\overline{q/A}$	-1		-1			
	Staniszewski	23	-1			-1		-1		1	
	Hatton and Hall	43	-2		T_s^2, D_c^{-2}	-4	2	-4	1	1	1
	Present author	Section 1.2	-2						-1	1	-1

Table 2. *Experimental results. Key to data points for Figs* 1, 2 *and* 7

Group	Author	Reference number	Liquid	% of $(q/A)_{cr}$	Symbol
Hydrodynamic	Cole	13	Water	~100	△
	Present results	Section 1.3	Water	~100	○
	Perkins and Westwater	29	Methanol	~100	⊖
Transition	Jakob and Linke	1	Water	<100	▲
	Jakob	16	Water	<100	◆
	Yamagata and Nishikawa	33	Water	<100	■
	Van Wijk and Van Stralen	24	Water	<100	◀
	Present results	Section 1.3	Water	<100	●
	Perkins and Westwater	29	Methanol	<100	●
	Westwater and Santangelo	34	Methanol	<100	▼
	Dunskus and Westwater	35	Isopropanol	<100	-●-
	Jakob and Linke	1	Carbon Tetrachloride	<100	▶
	Bobrovich and Mamontova	47	Water	15, 30, 45, 60	×
Thermodynamic	McFadden and Grassmann	17	Nitrogen	$\leqslant$100	▽
	Hatton and Hall	43	Water	5	□

in all the experiments reported, except those of Van Wijk and Van Stralen [24]. Thus in all other experiments the boiling condition was controlled by the heat flux (or temperature difference) for a given fluid, thermodynamic state and system condition, while the values assumed by f and D were dependent upon the values of these parameters. In Van Wijk and Van Stralen's experiment, an a.c. heated thin wire produced bubbles at a fixed frequency of 50 bubbles per second (sic.), thus frequency was an independent variable and bubble diameter was a dependent variable. It is of interest to note (see Fig. 1) that Van Wijk and Van Stralen's results for f and D are, nonetheless, similar to those in which f was a dependent variable.

To supplement the data from the literature, experiments were conducted in boiling from thin wires at atmospheric pressure and from the outside of tubes at sub-atmospheric pressures to provide further data on small and large bubbles respectively. The tests were made in a pool of demineralised water contained in a specially designed tank which could be subjected to vacuum. Using a Fastax WF4 camera, high speed cine films were taken of various boiling conditions to provide a variety of bubble diameters and corresponding bubble emission frequencies. The present results are compared with those from the literature in Fig. 2. Data on steam bubble rise velocities were also obtained, and these are shown in Fig. 6.

The hydrodynamic dimensionless group ($fD^{\frac{1}{2}}/g^{\frac{1}{2}}$) has been plotted in Fig. 2 against bubble diameter D. This group has been proposed by McFadden and Grassmann as providing a good correlation over all bubble diameters, and their correlation would be given by a horizontal line ($fD^{\frac{1}{2}}/g^{\frac{1}{2}}$) = 0·56. The figure shows that the vertical scatter of ($f\ D^{\frac{1}{2}}/g^{\frac{1}{2}}$) for all the experimental data cannot be reduced from that shown, unless the scatter for individual sets of data is reduced. This is because the scatter for separate sets of data is as large as the scatter for all data. Individual data may be examined for self-consistency by drawing smooth curves through each set of data. This has been done in Fig. 1, where the nitrogen data of McFadden and Grassmann and the water data of Hatton and Hall are fitted by $f \propto D^{-2}$. Similarly, the intermediate bubble diameter data for a number of investigations is fitted by $f \propto D^{-\frac{3}{4}}$, and the large bubble diameter data by $f \propto D^{-\frac{1}{2}}$. Thus different frequency–diameter relationships may

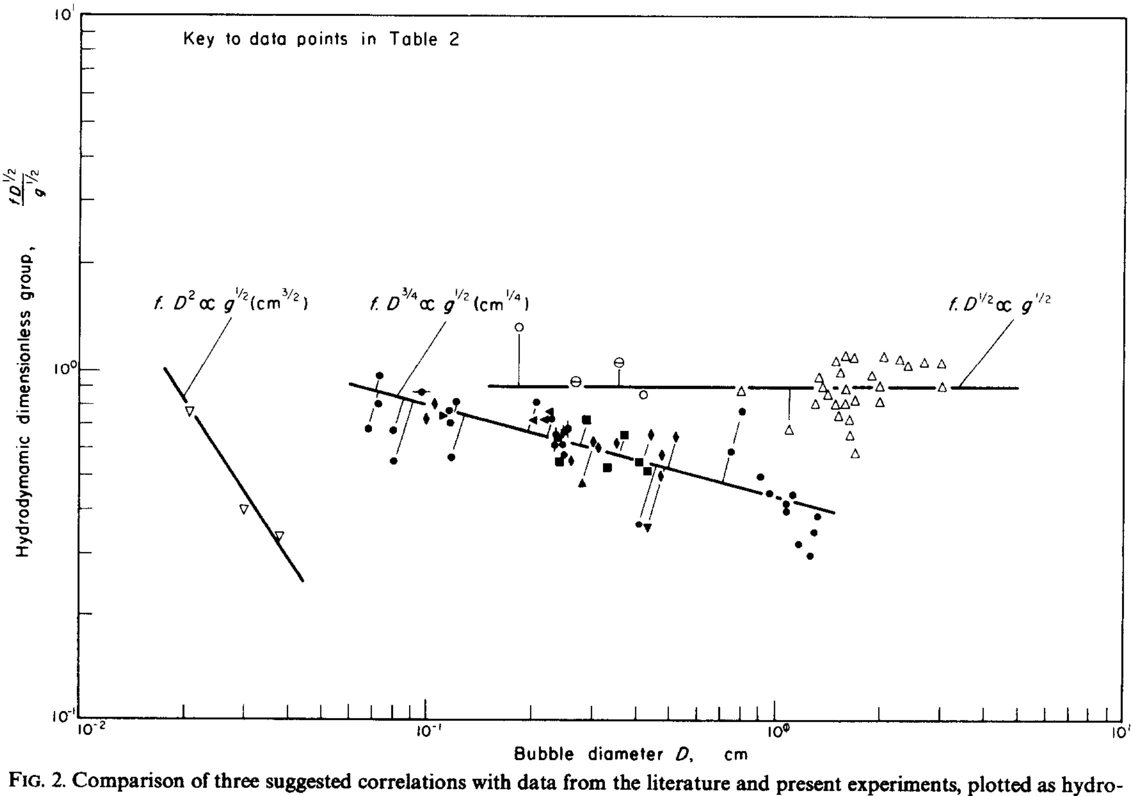

FIG. 2. Comparison of three suggested correlations with data from the literature and present experiments, plotted as hydrodynamic dimensionless group vs. bubble diameter.

arise as a result of the operation of different boiling mechanisms for different sets of experimental data.

The McFadden and Grassmann expression $(fD^{\frac{1}{2}}/g^{\frac{1}{2}}) = 0{\cdot}56$ provides a very approximate relationship between f and D over the whole range of D for which results are available. This expression could therefore be used as a rough approximation when details of the boiling process (range of bubble diameters, heat flux levels etc.) are unknown. It would, however, be correct to employ one of the more accurate expressions where details of the boiling process are known, or may be readily estimated. Referring to Table 1, the expressions relating f with D fall into two theoretical regions. The models may be based upon a hydrodynamic or a thermodynamic criterion, and between these a transition region exists. As might be expected, no very sharp distinctions may be made between the boiling regions appropriate to the three expressions. Some guidance on this is suggested for each of the regions:

(a) *Hydrodynamic region.* All models relating f with D in this region are similar, based as they are on equality of buoyancy and drag-forces. Bubble frequency is related to bubble diameter by inspection of Fig. 2 as:

$$fD^{\frac{1}{2}} = 0{\cdot}90g^{\frac{1}{2}}. \tag{22}$$

This applies to large bubbles $(D > 0{\cdot}5\text{ cm})$ at medium and high heat fluxes $(q/A)/(q/A)_{cr} >$ 20 per cent. It also applies to medium diameter bubbles $(0{\cdot}1 < D < 0{\cdot}5\text{ cm})$ at high heat fluxes $(q/A)/(q/A)_{cr} > 80$ per cent, where drag and buoyancy are the dominant forces [13].

Some evidence for this may be obtained from the experimental observations of Bobrovich and Mamontova [47] in pool boiling from a horizontal stainless steel strip 0·2 cm wide in water. Values of f and D were obtained at four heat fluxes. At the lowest heat flux $(q/A)/(q/A)_{cr} \simeq$ 15 per cent, the experimental point (X) falls in the transition region (see Fig. 1). The values at the three higher heat fluxes $(q/A)/(q/A)_{cr} \simeq 30$, 45, 60 per cent fall on the curve for the hydrodynamic region.

(b) *Transition region.* A number of limited experimental observations of f and D have been reported, and are shown in Table 1 where the exponent of D ranges from -1 to -3. Combining these with the present results, a different relationship is obtained. This has general applicability for medium bubble diameters (ranging from $D \simeq 0{\cdot}05$ cm at high heat fluxes, through $D \simeq 1$ cm at low heat fluxes). Thus f and D are related by experiment for the transition region, in which surface tension, drag and buoyancy forces are of the same order of magnitude:

$$fD^{\frac{3}{4}} = 0{\cdot}44\, g^{\frac{1}{2}}\,(\text{cm}^{\frac{1}{4}}). \tag{23}$$

The magnitudes of surface tension, drag and buoyancy forces acting on a spherical bubble in water in the bubble diameter region $10^{-2} \leqslant D \leqslant 10^{1}$ (cm) are shown in Fig. 3. The intersections of buoyancy and surface tension forces for methanol, isopropanal, carbon tetra-chloride and nitrogen are also shown. The intersections all occur in the range of diameter $0{\cdot}2 < D < 0{\cdot}5$ (cm). Comparison with Fig. 2 supports the view that it is a transition region, where surface tension, drag and buoyancy forces are of the same order of magnitude.

The above remarks form an approximate basis, being limited to spherical bubbles in equilibrium. Recent work has shown [48] that the relative magnitude of bubble forces (inertia and surface tension) during growth on the heater wall differs considerably according to the shape of the bubble. Thus surface tension was found to be the dominant force in spherical bubbles and the inertial force was dominant in hemispherical bubbles. For oblate bubbles, the inertial force was predominant in early growth, but less than the surface tension force in late growth of the bubble. The importance of drag force, however, was not considered, the analysis being based on Rayleigh's equation (as reported by Westwater [49]), in which the viscous force term was assumed negligible. As a bubble reaches its terminal rise velocity at the instant of departure [13], the drag force would be considerable for large bubbles, and should not be omitted from bubble growth studies.

Frederking and Daniels [50] have studied f and D for saturation film boiling from a 0·25 in dia. sphere in liquid nitrogen, and their results lie (without any preferred direction) in the region in Fig. 1 bounded by $20 < f < 40\,\mathrm{s}^{-1}$, and $0{\cdot}3 < D < 0{\cdot}6$ cm. Thus film boiling results might be included in the present scheme to relate f with D, but further work is required on this aspect. Frederking and Daniels also present some evidence that the dimensionless group $fD^{\frac{1}{2}}/g^{\frac{1}{2}}$ increases with increase in heat flux. Following critical discussion of their paper by Lienhard and Huang [51], the authors state that $fD^{\frac{1}{2}}$ is not constant but affected both by heat flux and bubble diameter. This is in accord with the present paper. The fact that bubble diameter is a function of heat flux increases the problem [47, 52].

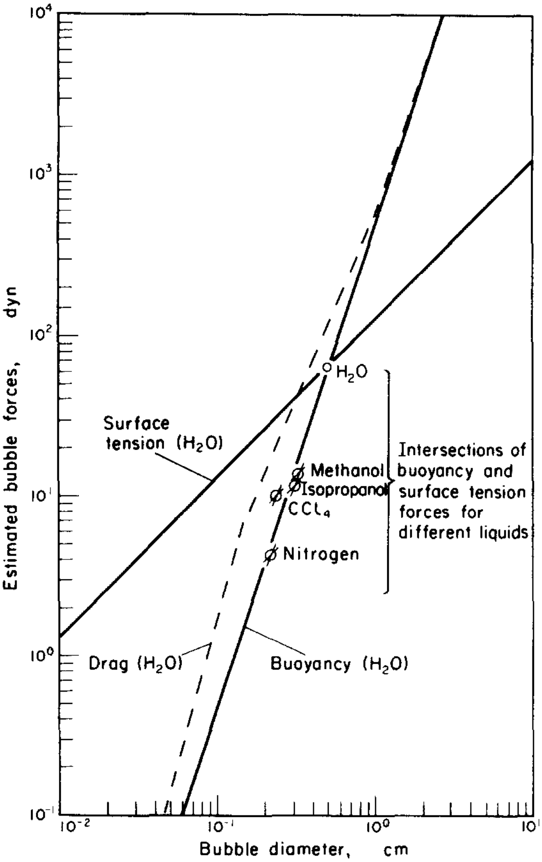

FIG. 3. Comparison of the absolute magnitudes of surface tension, drag and buoyancy forces for spherical bubbles in water.

(c) *Thermodynamic region.* Two of the theoretical expressions invoking thermodynamic considerations predict fD, and two others predict fD^2 (see Table 1). By inspection of Fig. 1, the nitrogen data of McFadden and Grassmann and the water data of Hatton and Hall fall on curves of the type $fD^2 =$ constant. Thermodynamic considerations arise in the case of small bubbles ($D \gtrsim 0{\cdot}05$ cm), as well as medium size bubbles ($0{\cdot}05 \gtrsim D \gtrsim 0{\cdot}5$ cm) at very low heat fluxes, where the frequency of bubble formation is largely governed by thermodynamic conditions during bubble growth. Variation of f and D is predicted correctly for the two sets of data both by the expression of Hatton and Hall, and of the present author.

2. THE RELATIONSHIP $u = fD$

2.1 *Introduction*

No single analytical relationship between f and D described in Section 1.2 has been accepted for general use in all the theories described in Section 1.1. To overcome this difficulty, a fictitious velocity term u has been used to replace the product fD, since it has the same dimensions. The term u is then assumed to be given by expressions for bubble rise velocities for single bubbles, i.e. Stoke's law [9], the drag–buoyancy force balance [7] etc.

In numerous theories dealing with the critical heat flux, a critical vapour velocity is associated with occurrence of the critical heat flux, whereby the vapour flow away from the heater surface relative to the liquid flow towards the surface is assumed to reach a critical value which may not be exceeded. Many theories equate this vapour velocity with a terminal bubble rise velocity, which strictly applies only for a single bubble in a large volume of liquid, as distinct from the chaotic behaviour of bubbles in the boiling process. It has never been demonstrated that the two radically different concepts of velocity (vapour velocity; bubble rise velocity) are in fact equivalent and interchangeable over the whole range of bubble diameters occurring in boiling.

Examples of theories for the critical heat flux which employ the above assumptions are given by: Griffith [9], in which the fD term was replaced by V_{growth} (an average bubble growth velocity); Rohsenow and Griffith [8], in which fD was equated to the bubble velocity; Zuber [11], in which the assumption $D/t_b = u$ was made for nucleate boiling, and $\lambda/\tau = u_{\text{critical}}$ for the critical heat flux where λ and τ are assumed to be bubble diameter and period respectively; and Deissler [7], in which the critical heat flux was assumed to occur when successive bubbles touched, that is $u = fD$.

Theories for the nucleate boiling region also employ similar assumptions, for example: Zuber [26], in which the expression $fD/u = 1$ served to ascertain the maximum possible frequency of successive bubbles emanating from a single site in the "laminar" region. The expression also served to give the bubble population corresponding to the maximum vapour hold-up in the laminar region, and was used in the turbulent region to determine the vapour hold-up; and Lienhard [27], in which fD was assumed to be the velocity at which bubbles rise for zero separation between successive bubbles. Zuber's [11] expression for fD served as the velocity term to determine the turbulent drag on the rising bubbles.

Present justification for the use of $u = fD$ is obtained from experimental observations illustrated in Fig. 4. The frequency of bubble emission at a single site may be written as:

$$f = \frac{1}{t_d + t_c}. \qquad (24)$$

For low heat fluxes, Jakob and Linke [1] found that $t_c \simeq t_d$. For medium heat fluxes, Donald and Haslam [28] and Perkins and Westwater [29] noted that $t_c = 0$. Thus for most cases $f \simeq 1/t_d$. It has also been noted that the bubble velocity is approximately constant as the bubble grows and leaves the heater surface [1, 13, 28]; from which $u \simeq D/t_d$. It then follows that $u \simeq fD$.

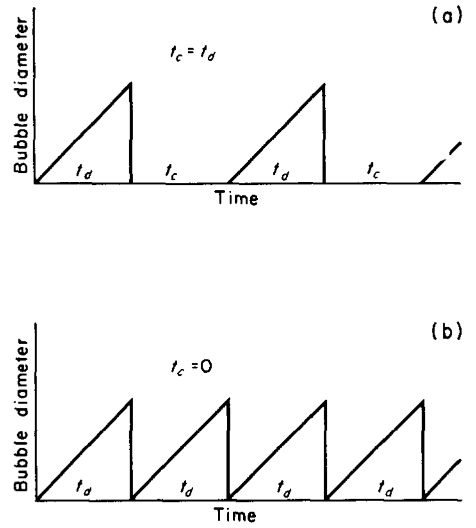

FIG. 4. Bubble behaviour at a nucleating site as a function of time.
t_c = contact time of cool liquid with heater surface,
t_d = nucleation and growth time of bubble on heater surface.
(a) Observations of Jakob and Linke [1].
(b) Observations of Donald and Haslam [28] and Cole [13].

Thus, the above theories have equated the product of frequency and bubble diameter occurring during the chaotic behaviour of the boiling process with the rise velocity for a single bubble in a large body of quiescent liquid. It has never been determined whether the two radically different concepts of velocity (bubble frequency times diameter; bubble rise velocity) are in fact equivalent and interchangeable over the whole range of bubble diameter which occurs in boiling.

2.2 *Comparison of the experimental value of bubble velocity (u) with the product (fD) over a range of bubble diameters*

To test the extent of the validity of the widely assumed relationship $u = fD$, the bubble rise velocities of both air and steam bubbles in water are plotted in Figs. 5 and 6 respectively as functions of bubble diameter. The data available from the literature for steam bubble rise velocities has been augmented with experimental observations by the present author. Bubble rise velocities for large diameter bubbles were obtained at sub-atmospheric pressures, and small diameter bubble rise velocities were obtained at atmospheric pressure. Figure 6 shows that the steam bubble rise velocity–diameter relationship (given by the data points) is very similar to that for air bubbles (given by the curves and obtained from the data in Fig. 5). In Fig. 7 the product fD(cm/s) is compared directly with the curves for air bubble rise velocity (cm/s) on a common abscissa of bubble diameter.

Although there is lack of detailed agreement between the two concepts, there is clearly a general correspondence between the two as a function of bubble diameter over the whole range of diameter shown. Much closer agreement is obtained between the two concepts when the hydrodynamic region of bubble diameter only is considered [$0{\cdot}13 < D < 10$ (cm)]. In Fig. 6 the steam bubble rise velocities u lie along and slightly above the curve for air bubble rise velocity, while in Fig. 7 the experimental points for fD for the hydrodynamic region only (for key see Table 2) also lie along and slightly above the air bubble curve. The close agreement between the product fD and bubble rise velocity u for the case of the hydrodynamic region is not surprising, as the arguments for equating the product fD to u have arisen from analytical hydrodynamic considerations of the boiling phenomenon.

It may therefore be concluded that the expression $u = fD$ does have some justification as an approximate quantitative identity as well

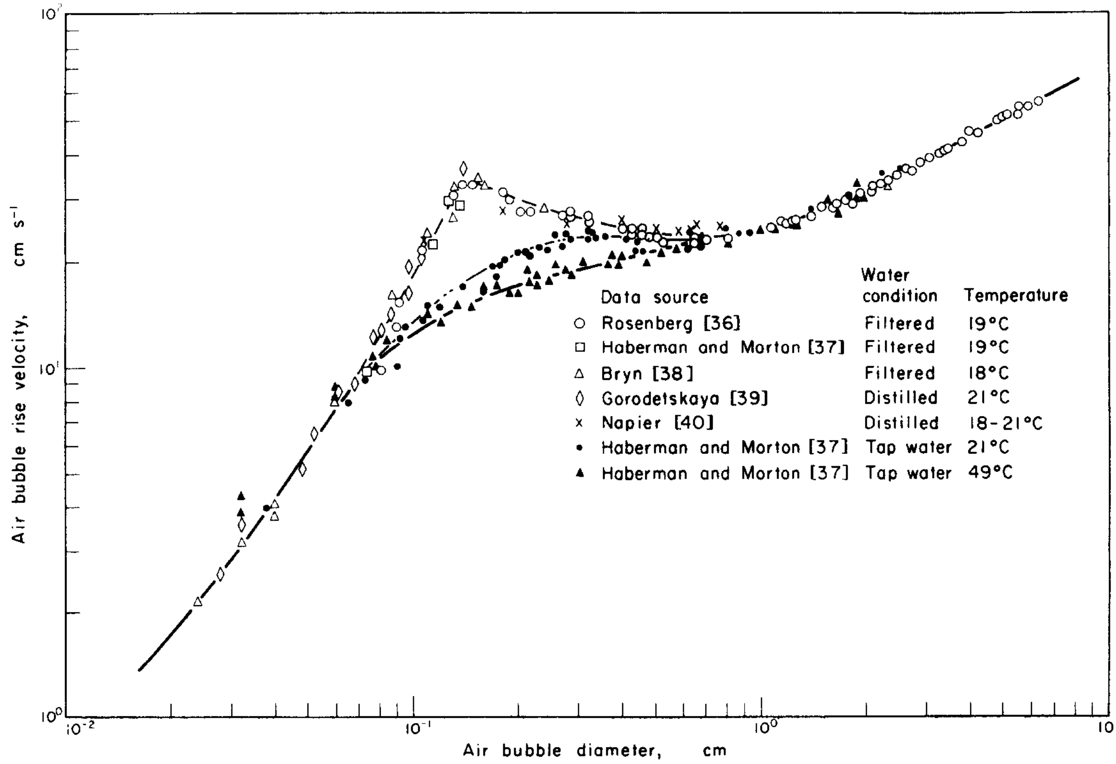

FIG. 5. Bubble rise velocity–diameter curves for single air bubbles in water.

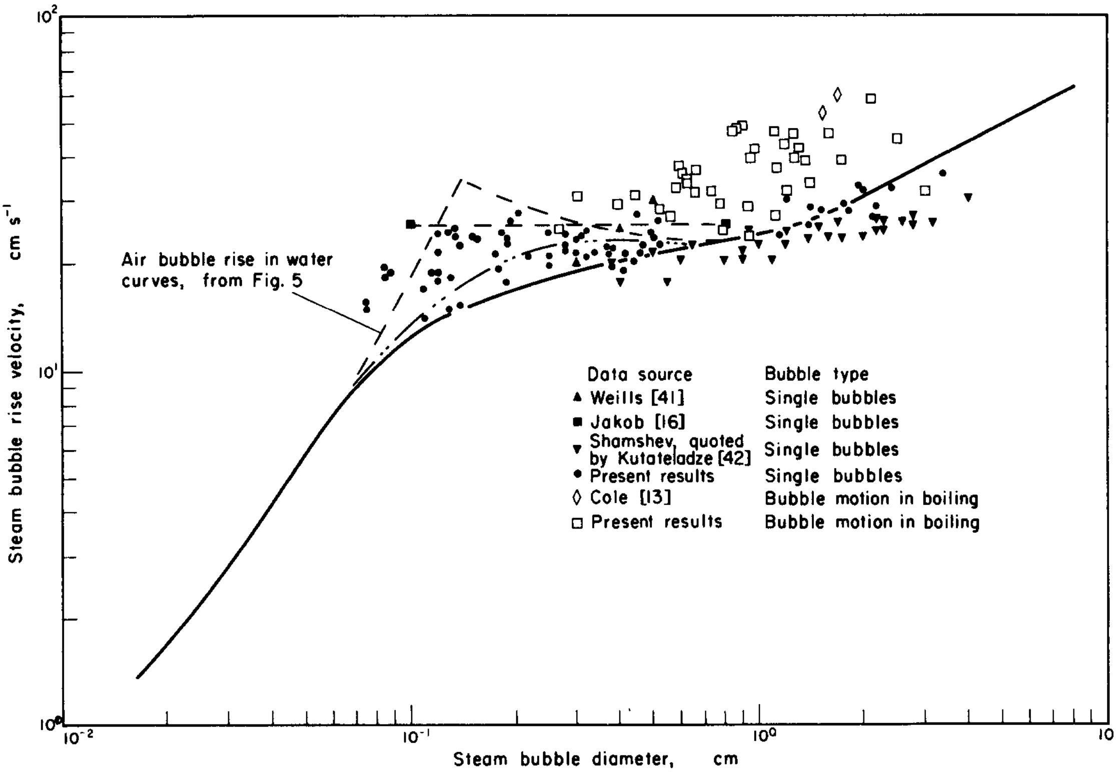

FIG. 6. Steam bubble rise velocity–diameter curves for single steam bubbles and boiling bubbles in water.

as an approximate dimensionless relationship. However, it should not be incorporated in accurate theoretical analyses of the boiling phenomenon, in which the proportionality constant relating dimensionless groups is obtained as an exact theoretical value, owing to the naturally occurring scatter of ± 50 per cent evident in Figs. 6 and 7.

3. CONCLUSIONS

3.1 *The relationship* $f = \phi(D \ldots)$

Evidence is given which indicates that a single expression may not adequately correlate bubble frequency with bubble diameter for all bubble diameters in nucleate boiling. Three separate regions are suggested with which the experimental data are better correlated:

(a) Hydrodynamic region, in which

$$fD^{\frac{1}{2}} = 0{\cdot}90\, g^{\frac{1}{2}},$$

(b) Transition region, in which

$$fD^{\frac{3}{4}} = 0{\cdot}44\, g^{\frac{1}{2}}\,(\mathrm{cm}^{\frac{1}{4}})$$

(c) Thermodynamic region, in which

$$fD^2 = \text{constant}.$$

Theoretical explanations for the hydrodynamic region and the thermodynamic region are proposed, which account for the experimental variation of bubble frequency with diameter in the two regions.

3.2 *The relationship* $u = fD$

The widely used identity $u = fD$, between bubble rise velocity and the product of bubble frequency and diameter, is demonstrated to be only approximately valid over the wide range of bubble diameters considered. There is, however, closer agreement between experimental

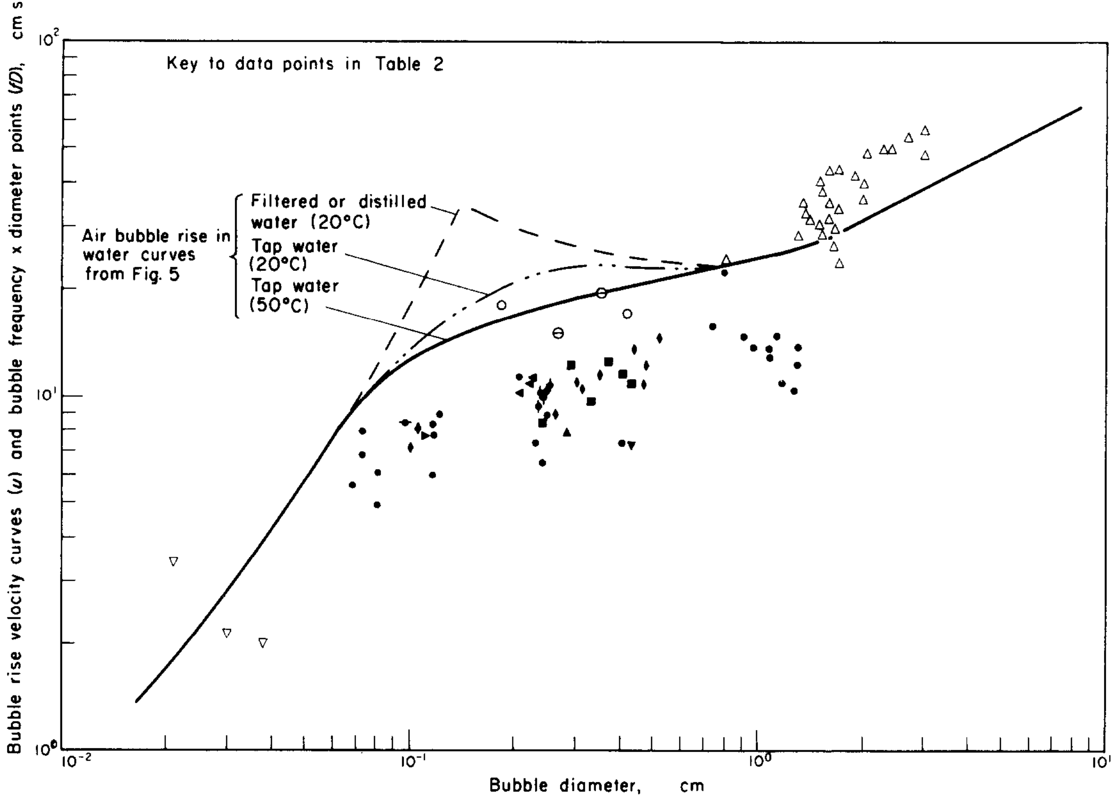

FIG. 7. Comparison of fD product with air bubble rise velocity–diameter curves from Fig. 5.

bubble rise velocities and frequency–diameter products over a range of bubble diameters in the hydrodynamic region. This is to be expected, since the arguments equating u and fD have arisen from hydrodynamic considerations of the boiling phenomena.

The relationship $u/fD = 1$ should not be incorporated into accurate theoretical analyses of the boiling phenomenon, in which the proportionality constant relating dimensionless groups is obtained as an exact theoretical value, because the experimental scatter indicates an approximate relationship only.

ACKNOWLEDGEMENTS

The experimental part of this investigation was undertaken during 1960–62, when the author was working in the Engineering Laboratories of the Atomic Energy Establishment at Winfrith. The author wishes to express his thanks to the UKAEA for permission to publish the results of these experiments.

REFERENCES

1. M. JAKOB and W. LINKE, *Heat Transfer*, Vol. 1, p. 642. John Wiley, New York (1949).
2. W. M. ROHSENOW, A method of correlating heat-transfer data for surface boiling of liquids, *Trans. Am. Soc. Mech. Engrs* **74**, 969 (1952).
3. L. S. STERMAN, On the theory of the heat transfer from a boiling fluid, *Z. Tekh. Fiz.* **23** (2), 341 (1953).
4. G. G. TRESHCHOV, Experimental investigation of the mechanism of heat transfer with surface boiling of water, *Teploenergetika* **4**(5), 44 (1957).
5. E. R. G. ECKERT, *Introduction to the Transfer of Heat and Mass*, 1st edn., p. 52. McGraw-Hill, New York (1950).
6. H. J. IVEY, Acceleration and the critical heat flux in pool boiling heat transfer, *Proc. Inst. Mech. Engrs.* **177** (1), 15 (1963).
7. R. G. DEISSLER, see paper by R. COLE, A photographic study of pool boiling in the region of the critical heat flux, *A.I.Ch.E. Jl* **6**(4), 533 (1960).
8. W. M. ROHSENOW and P. GRIFFITH, Correlation of maximum heat flux data for boiling of saturated liquids, *Chem. Engng Prog. Symp. Ser.* **52**(18), 47 (1956).
9. P. GRIFFITH, The correlation of nucleate boiling burnout data, A.S.M.E. Paper No. 57-HT-21 (1957).
10. Y. P. CHANG and N. W. SNYDER, Heat transfer in

saturated boiling, *Chem. Engng Prog. Symp. Ser.* **56**(30), 25 (1960).

11. N. Zuber, Hydrodynamic aspects of boiling heat transfer, A.E.C.U. Report No. 4439 (1959).
12. C. J. Rallis and H. H. Jawurek, Latent heat transport in saturated nucleate boiling, *Int. J. Heat Mass Transfer* 7, 1051 (1964).
13. R. Cole, A photographic study of pool boiling in the region of the critical heat flux, *A.I.Ch.E. Jl* **6**(4), 533 (1960).
14. F. N. Peebles and H. J. Garber, Studies on the motion of gas bubbles in liquids, *Chem. Engng Prog.* **49**(2), 88 (1953).
15. E. P. Wigner, The rate of rise of air bubbles, A.E.C.D. Report No. 1983 (1948).
16. M. Jakob, *Heat Transfer*, Vol. 1, p. 643. John Wiley, New York (1949).
17. P. W. McFadden and P. Grassmann, The relation between bubble frequency and diameter during nucleate pool boiling, *Int. J. Heat Mass Transfer* **5**, 169 (1962).
18. R. M. Davies and G. I. Taylor, The mechanics of large bubbles rising through extended liquids and through liquids in tubes, *Proc. R. Soc.* **A200**, 375 (1950).
19. K. Nishikawa and K. Urakawa, An experiment of nucleate boiling under reduced pressure, *Trans. Japan. Soc. Mech. Engrs* **23**, 935 (1957).
20. R. F. Gaertner and J. W. Westwater, Population of active sites in nucleate boiling heat transfer, *Chem Engng Prog. Symp. Ser.* **56**(30), 39 (1960).
21. R. Siegel and E. G. Keshock, Effects of reduced gravity on nucleate boiling bubble dynamics in saturated water, *A.I.Ch.E.Jl* **10**(4), 509 (1964).
22. L. M. Zizina-Molozhen and S. S. Kutateladze, The influence of pressure on the mechanism of steam formation in a boiling liquid, *Z. Tekh. Fiz.* **20**(1), 110 (1950).
23. B. E. Staniszewski, Nucleate boiling bubble growth and departure, M.I.T. Division of Industrial Cooperation Technical Report No. 16 (1959).
24. W. R. Van Wijk and S. J. D. Van Stralen, Growth rate of vapour bubbles in water and in a binary mixture boiling at atmospheric pressure, *Physica's Grav.* **28**, 150 (1962).
25. P. Griffith, Bubble growth rates in boiling, *Trans. Am. Soc. Mech. Engrs* **24**, 721 (1958).
26. N. Zuber, Nucleate boiling. The region of isolated bubbles and the similarity with natural convection, *Int. J. Heat Mass Transfer* **6**, 53 (1963).
27. J. H. Lienhard, A semi-rational nucleate boiling heat flux correlation, *Int. J. Heat Mass Transfer* **6**, 215 (1963).
28. M. B. Donald and F. Haslam, The mechanism of the transition from nucleate to film boiling, *Chem. Engng Sci.* **8**, 287 (1958).
29. A. S. Perkins and J. W. Westwater, Measurements of bubbles formed in boiling methanol, *A.I.Ch.E. Jl* **2**(4), 471 (1956).
30. E. Buckingham, Model experiments and the forms of empirical equations, *Trans. Am. Soc. Mech. Engrs* 263–296 (1915).
31. C. M. Usiskin and R. Siegel, Experimental study of boiling in reduced and zero gravity fields, Trans. A.S.M.E. Paper No. 60-HT-10 (1960), and *J. Heat Transfer* **83C**, 243 (1961).
32. R. Siegel and C. M. Usiskin, A photographic study of boiling in the absence of gravity, *J. Heat Transfer* **81C**, 230 (1959).
33. K. Yamagata and K. Nishikawa, Nucleate boiling of water on the horizontal heating surface, *Mem. Fac. Engng, Kyushu Univ.* **15**, 1 (1955).
34. J. W. Westwater and J. G. Santangelo, Photographic study of boiling, *Ind. Engng Chem.* **47**(8), 1605 (1955).
35. T. Dunskus and J. W. Westwater, The effect of trace additives on the heat transfer to boiling isopropanol, Fourth National Heat Transfer Conference (U.S.A.), A.I.Ch.E. Preprint 30 (1960).
36. B. Rosenberg, The drag and shape of air bubbles moving in liquids, Taylor Model Basin Report TMB 727 (1950).
37. W. L. Haberman and R. K. Morton, An experimental investigation of the drag and shape of air bubbles rising in various liquids, Taylor Model Basin Report TMB 802 (1953).
38. T. Bryn, Speed of rise of air bubbles in liquids, Taylor Model Basin Translation TMB 132 (1949).
39. A. Gorodetskaya, The rate of rise of bubbles in water and aqueous solutions at great Reynolds numbers, *J. Phys. Chem. (USSR)* **23**, 71 (1949).
40. D. H. Napier, Ascent of air bubbles in air-saturated water, Imperial College (London University) Dept. Chem. Engng Gas Absorption Working Party Report No. 10 (1949).
41. J. T. Weills, Steam bubble size and rate of rise through water at its normal boiling point, Report No. CT-910 (1943).
42. S. S. Kutateladze, U.S.A.E.C. Report No. AEC-tr-3770, p. 89 (1959).
43. A. P. Hatton and I. S. Hall, Photographic study of boiling on prepared surfaces, Paper No. 115, Third International Heat Transfer Conference, Chicago (1966).
44. M. Plesset and S. A. Zwick, The growth of vapour bubbles in superheated liquids, *J. Appl. Phys.* **25**, 493 (1954).
45. Y. Y. Hsu, On the size range of active nucleation cavities on a heating surface, *J. Heat Transfer* **84C**, 207 (1962).
46. C-Y. Han and P. Griffith, The mechanism of heat transfer in nucleate pool boiling, Part I—Bubble initiation, growth and departure. *Int. J. Heat Mass Transfer* **8**, 887 (1965).
47. G. I. Bobrovich and N. N. Mamontova, A study of the mechanism of nucleate boiling at high heat fluxes, *Int. J. Heat Mass Transfer*, **8**, 1421 (1965).
48. M. A. Johnson, J. de la Pena and R. B. Mesler, Bubble shapes in nucleate boiling, *Chem. Engng Prog. Symp. Ser.* **62**(64), 1 (1966).
49. J. W. Westwater, Boiling heat transfer, *Am. Scient.* **47**, 427 (1959). 1 (1966).
50. T. H. K. Frederking and D. J. Daniels, The relation between bubble diameter and frequency of removal from a sphere during film boiling, *J. Heat Transfer* **88C**, (1), 87 (1966).
51. J. H. Lienhard and C. P. Huang, Discussion of paper

by T. H. K. FREDERKING and D. J. DANIELS, The relation between bubble diameter and frequency of removal from a sphere during film boiling, *J. Heat Transfer* **88C**(1), 91 (1966).

52. H. J. IVEY, The variation of bubble departure diameter with pressure in nucleate pool boiling, Paper in preparation.

APPENDIX 1

Bubble Rise Velocity Expression of Peebles and Garber

Peebles and Garber [14] employed a relationship for u, first suggested by Wigner [15], for the region of bubble diameter in which bubble rise velocity is approximately independent of bubble diameter:

$$u = 1{\cdot}18\left(\frac{g g_c \sigma}{\rho_L}\right)^{\frac{1}{4}}.$$

Wigner assumed that the bubble rise velocity u was independent of diameter for continuously growing bubbles, and that $u = \phi(\rho_L, \sigma, g, \mu_L)$. Dimensional analysis together with experimental results gave:

$$\frac{u\mu_L}{\sigma g_c} = 2\left(\frac{\rho_L \sigma^3 g_c^3}{\mu_L^4 g}\right)^{-\frac{1}{4}}.$$

From which:

$$u = 2\left(\frac{\sigma g g_c}{\rho_L}\right)^{\frac{1}{4}}$$

thus indicating that bubble rise velocity was independent of viscous forces.

APPENDIX 2

Dimensional Analysis Applied to the Bubble Diameter–Frequency Problem in the Hydrodynamic Region

McFadden and Grassmann [17] used dimensional analysis to obtain a relationship between f and D by assuming that fD is a function of D, σ, ρ_L and $\Delta\rho$; but in the two dimensionless groups so obtained, the gravity term g appeared, indicating that the method of dimensional analysis had not been applied rigorously.

The present analysis attempts to rationalise the problem in terms of hydrodynamic factors (an unstated but implicit assumption in McFadden and Grassmann's analysis). The diameter and frequency of vapour bubbles are here assumed to depend on the forces acting on the bubble, namely buoyancy, drag, surface tension, inertia, and viscosity.

The viscous forces will be neglected, as Wigner [15] found bubble rise velocity was independent of viscous forces for bubble sizes occurring in saturation boiling. The functional relationship given by the above assumptions is then

$$f = \phi(D, \Delta\rho, g, \rho_L, u, \sigma, \rho_v, \dot{u}).$$

According to Buckingham [30], three dimensions and nine parameters will give six dimensionless groups, from which the following relationship is obtained using the Rayleigh dimensional method

$$\frac{\rho_L u^2 D^2}{g\Delta\rho D^3} = \phi\left(\frac{g_c \sigma D}{g\Delta\rho D^3}, \frac{fD}{u}, \frac{\rho_L}{\rho_v}, \frac{\dot{u}}{g}, \frac{\Delta\rho}{\rho_L}\right).$$

The first two dimensionless groups are the ratios of drag to buoyancy forces, and surface tension to buoyancy forces. This arrangement was chosen because the dominant force which tears bubbles away from the heater surface (in saturation pool boiling) is the buoyancy force. This has been demonstrated by Usiskin and Siegel [31, 32] under near-zero-g boiling conditions.

For most conditions the approximation $\Delta\rho \simeq \rho_L$ holds. It has also been observed experimentally by Jakob [16] and Cole [13] that steam bubbles rise with an approximately uniform velocity at departure from the heating surface; thus $\dot{u} \simeq 0$. Finally, assuming that bubbles leave with no waiting period (for moderate heat fluxes) at departure [28], and rise with uniform velocity as indicated above, then

$u \simeq fD$. These assumptions give

$$\frac{f^2D}{g} = \phi\left(\frac{g_c\sigma}{g\Delta\rho\, D^2}, \frac{\rho_L}{\rho_v}\right).$$

The above expression contains the term (ρ_L/ρ_v), which is additional to those obtained by McFadden and Grassmann. In their analysis the term ρ_v was excluded, and as for most conditions $\rho_L \simeq \Delta\rho$, the group (ρ_L/ρ_v) was eliminated from their analysis by their initial assumptions.

Résumé—De nombreuses theories du transport de chaleur par ébullition font rentrer la fréquence f, le diamètre D et la vitesse de montée u des bulles dans leurs expressions du flux de chaleur d'ébullition nucléée et du flux de chaleur critique. Afin de simplifier ces expressions, on emploie souvent des relations individuelles entre f et D, et la relation $u = fD$, dans toute la gamme des diamètres de bulles. On met en évidence une relation unique entre f et D pour n'importe quel diamètre de bulle, mais seulement d'une façon approchée. Les résultats expérimentaux se divisent en trois régions distinctes dans lesquelles ils sont représentés mieux que par une relation unique: (a) Région hydrodynamique, dans laquelle les forces principales agissent sur la bulle sont la force de flottaison et la traînée: $fD^{\frac{1}{2}} = 0{,}90\ g^{\frac{1}{2}}$; (b) Région de transition, dans laquelle la force de flottaison, la traînée et la tension superficielle sont du même ordre: $fD^{\frac{3}{4}} = 0{,}44\ g^{\frac{1}{2}}$ (cm$^{\frac{1}{4}}$); (c) Région thermodynamique, dans laquelle prédominent les conditions existant pendant la croissance de la bulle: $fD^2 = \mathrm{C}^{\mathrm{te}}$. Des explications théoriques pour la variation de f avec D sont proposées pour les régions (a) et (c).

Les observations expérimentales indiquent que la relation $u = fD$ est valable seulement d'une façon approchée dans la gamme des diamètres de bulles: $0{,}02 > D > 3$ cm.

La relation $u/fD = 1$ ne devrait donc pas être prise en compte dans des analyses théoriques précises de l'ébullition, dans lesquelles la relation de proportionnalité entre des groupes sans dimensions est obtenue théoriquement d'une façon exacte.

Zusammenfassung—Viele Analysen des Wärmeübergangs beim Sieden führen in ihren Ausdrücken für die Wärmestromdichte beim Sieden und bei der Siedekrisis die Blasenfrequenz f an, den Blasendurchmesser D und die Blasensteiggeschwindigkeit u. Zur Vereinfachung dieser Ausdrücke werden oft besondere Beziehungen zwischen f und D, und die Beziehung $u = fD$ für den gesamten Bereich der Blasendurchmesser verwendet.

Es wird bewiesen, dass eine einzelne Beziehung für alle Blasendurchmesser beim Sieden die Werte f und D nur näherungsweise zueinander in Beziehung setzen kann. Auf Grund von Versuchsdaten ergeben sich drei getrennte Bereiche, wobei die Daten der einzelnen Bereiche durch gesonderte Gleichungen besser beschrieben werden können, als dies in einer einzigen Beziehung möglich ist: (a) Hydrodynamischer Bereich, in welchem Auftrieb und Widerstand die überwiegend auf die Blase wirkenden Kräfte sind: $fD^{\frac{1}{2}} = 0{,}90\ g^{\frac{1}{2}}$; (b) Übergangsbereich, in dem Auftriebs-, Widerstands- und Oberflächenspannungskräfte vergleichbar sind: $fD^{\frac{3}{4}} = 0{,}44\, g^{\frac{1}{2}}$ (cm$^{\frac{1}{4}}$); (c) Thermodynamischer Bereich, in dem die Bedingungen während des Blasenwachstums vorherrschen $fD^2 = \mathrm{const.}$

Für die Bereiche (a) und (c) werden theoretische Erklärungen für die Änderung von f mit D angeführt. Beobachtungen beim Versuch ergeben, dass die Beziehung $u = fD$ nur für einen Blasendurchmesser von $0{,}02 > D > 3$ cm näherungsweise gültig ist. Deshalb sollte die Beziehung $u/fD = 1$ nicht in genaue theoretische Analysen des Siedens eingesetzt werden, bei welchen die Proportionalkonstante dimensionsloser Gruppen als exakter theoretischer Wert erhalten wird.

Аннотация—Многие анализю по теплообмену при кипении включают величины: частота пузырьков f диаметр цузырьков D и скорость подъема u в выражениях для теплового потока при пузырьковом кипении и для критического теплового потока. Для упрощения этих уравнений часто используются отдельные зависимости между f и D и выражение $u = fD$ во всем диапазоне изменения диаметров пузырьков.

Подтверждается, что единственная зависимость только приблизительно увязывает f и D для всех диаметров пузырьков при пузырьковом кипении. Исходя из экспериментальных данных, предложено три соотношения для трёх отдельных областей, которые гораздо лучше увязывают данные для каждой области, чем единственная зависимость: (а) Гидродинамическая область, в которой основными силами, влияющими на пузырёк, являются подъемная сила и сопротивление: $fD^{\frac{1}{2}} = 0{,}90\, g^{\frac{1}{2}}$; (б) Переходная область, в которой сравнимы по величине подъёмная сила, сопротивление и сила поверхностного натяжения: $fD^{\frac{3}{4}}\ 0{,}44\, g^{\frac{1}{2}}$ (см$^{\frac{1}{4}}$); (в) Термодинамическая область, в которой условия в период роста пузырьков являются предоминирующими: $fD^2 =$

constant. Предлагается теоретическое объяснение изменения f с изменением D для областей (а) и (в).

Экспериментальные исследования указывают, что соотношение $u = fD$ только приблизительно справедливо в диапазоне изменения диаметров пузырьков: $0{,}02 > D > 3$ см.

Поэтому соотношение $u/fD = 1$ не должно включаться в точные теоретические анализы явления кипения, в которых пропорциональность постоянного соотношения безразмерных групп получается как *точная* теоретическая величина.